
Mathematics Calculations Policy

September 2021

To be reviewed Summer 2023

Mathematics Calculation Policy

Introduction and rationale

Children are introduced to the processes of calculation through practical, oral, mental and written activities. As they begin to understand the underlying ideas they develop ways of recording their thoughts – they move from informal to more formal methods of recording.

Children make this journey by using manipulatives (counting sticks, Deines' Apparatus, number beads etc), models and images (such as empty number lines, times table square etc) and a gradual appreciation of the need for clear working out in order to achieve correct answers. Children are encouraged not only to simply carry out a calculation, but, perhaps more importantly, are taught to explain what they are doing and why they are doing it.

Children in all classes are taught to think about the practical uses of calculation and maths in general in the “real world”.

By the end of Year 6 most children will be equipped with mental, written (both formal and informal / jottings) that they understand and can use correctly. Some children are also taught how to use a calculator. When faced with a calculation, children are taught to make an estimate first, then to solve the calculation using whichever method is most appropriate. They are then taught to check that the answer is reasonable.

At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts. Our “Home learning Policy” makes parents aware that the earlier the “building blocks” of number bonds and times tables can be embedded, the easier the children will find their work at school.

The overall aim is that when children leave St Nicholas at Wade, they:

- have a secure knowledge of number facts and a good understanding of the four number operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations, by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another (particularly the usefulness of the “inverse” operation) and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas and forms an important part of the majority of maths lessons.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers and patterns comes from regular practice and repetition.

Secure mental calculation requires the ability to:

- recall key multiplication facts instantly - confidence with the 2, 5 and 10 times tables by the end of Year 2, the 2, 3, 4, 5, 8 and 10 times tables by the end of Year 3 and all multiplication facts up to 12×12 by the end of Year 4;
- learning and applying addition and subtraction facts - all addition and subtraction facts for each number to 20 by the end of Year 1, adding a 3 digit number and 1s or 10s by the end of Year 3 and solving increasingly large numbers (such as $12462 - 2300$) by the end of Year 5;
- use taught strategies to work out the calculation - recognise that addition can be done in any order and use this to add mentally two 2-digit numbers by the end of Year 2, to be able to partition numbers, add tens and units separately, then put back together to achieve the answer (for example $63 + 28 = 60 + 20 = 80$, $3 + 8 = 11$, $80 + 11 = 91$) by the end of Year 3.

Written methods of calculation

The aim is that by the end of Key Stage 2, the majority of children should be able to use an efficient written method for each operation with confidence and understanding. They should also be able to explain what they are doing, and why. Children will begin each type of calculation using an expanded method which builds firmly on their mental ability and mathematical understanding. These strategies will be developed into more efficient, compact methods.

Children need to be thoroughly secure in a range of mental strategies before they are ready to begin to use more formal written methods. Each teacher has the responsibility to ensure that learning is reinforced and that children move progressively from informal to more formal methods of recording when they are able understand and verbalise the mathematical process.

This document show the methods developed for each of the four rules. There is a guide to which year the majority of children in the year group would be expected to develop the method, but as children develop and learn at different rates, there may be children working at different levels.

The layout for each calculation type will rarely vary during lessons. This means that parents can then have the confidence that the methods they are teaching their children at home mirror the methods being taught in school.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Addition and Subtraction

Vocabulary

Abstract, base ten, calculation, concrete, counters, decrease, difference, equal, exchange, fewer, hundreds, less, minus, multilink, numicon, number line, number track, ones, partition, pictorial, place value, PPW – part-part-whole, record, subtract, subtrahend, take away, tens, thousands, total

National Curriculum - Statutory Requirements

Year One – Pupils should be taught to:-

- * read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- * represent and use number bonds and related subtraction facts within 20
- * add and subtract one-digit and two-digit numbers to 20, including zero
- * solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \quad - 9$.

Year Two – Pupils should be taught to:-

- * solve problems with addition and subtraction:
 - * using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - * applying their increasing knowledge of mental and written methods
- * recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- * add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- * show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- * recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Year Three – Pupils should be taught to:-

- * add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- * add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- * estimate the answer to a calculation and use inverse operations to check answers
- * solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Year Four – Pupils should be taught to:-

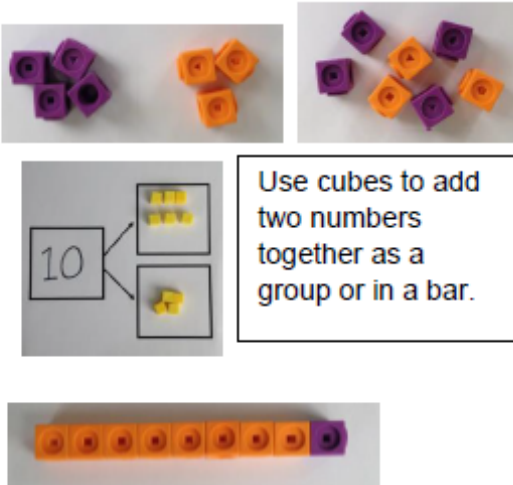
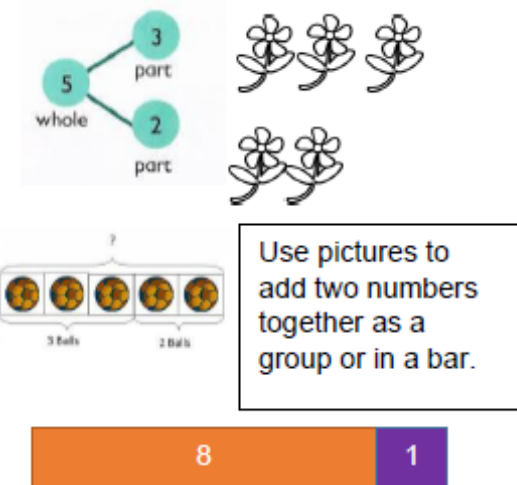


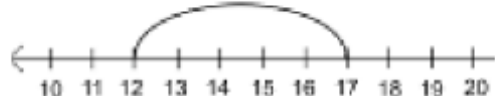
- * add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- * estimate and use inverse operations to check answers to a calculation
- * solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Year Five – Pupils should be taught to:-

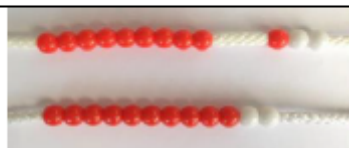
- * add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- * add and subtract numbers mentally with increasingly large numbers
- * use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- * solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Year Six – Pupils should be taught to:-

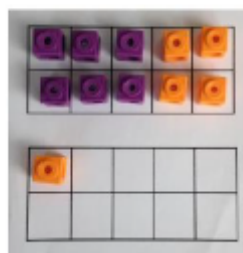
- * use their knowledge of the order of operations to carry out calculations involving the four operations
- * solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- * solve problems involving addition, subtraction, multiplication and division
- * use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

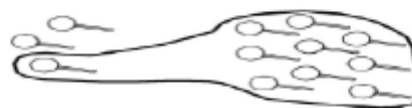
Regrouping to make 10.



$$6 + 5 = 11$$

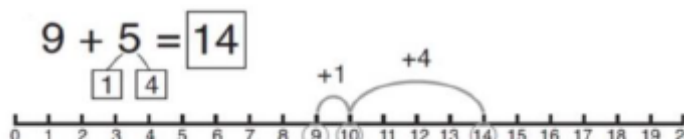


Start with the bigger number and use the smaller number to make 10.



$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number to make 10.



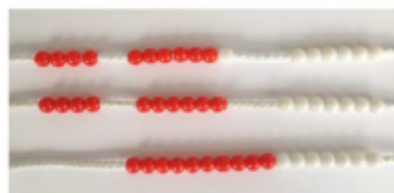
$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

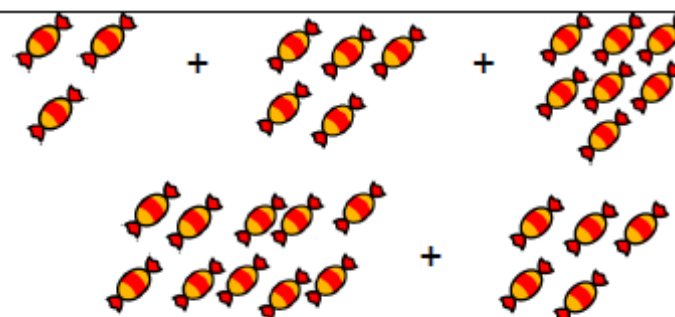
Adding three single digits

$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

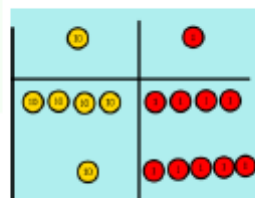
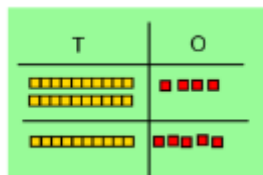
$$\begin{array}{l} (4 + 7 + 6) = [10] + [7] \\ \quad \quad \quad 10 \\ \quad \quad \quad = [17] \end{array}$$

Combine the two numbers that make 10 and then add on the remainder.

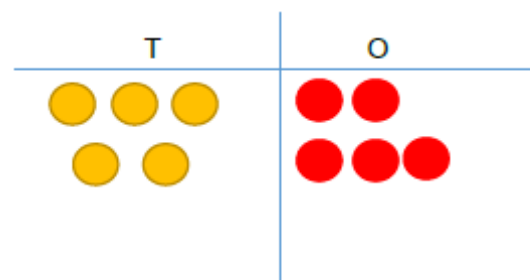
Column method- no regrouping

$$24 + 15 =$$

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



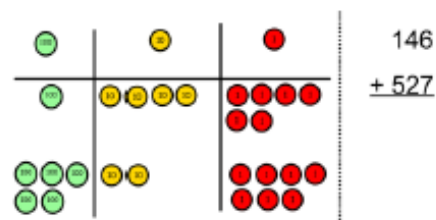
Calculations

$$21 + 42 =$$

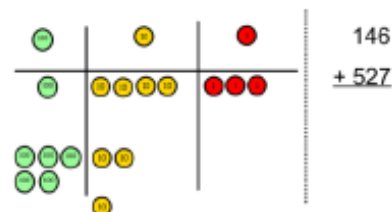
$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

Column method- regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

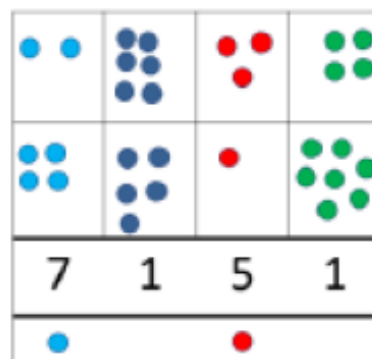


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

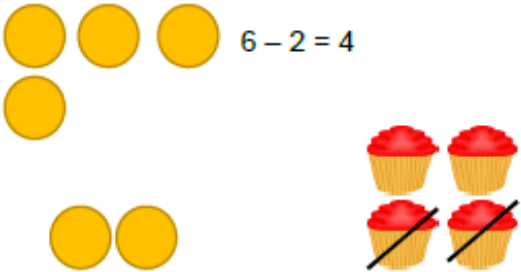
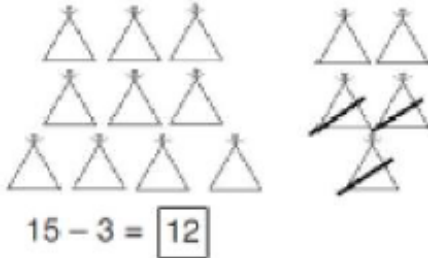


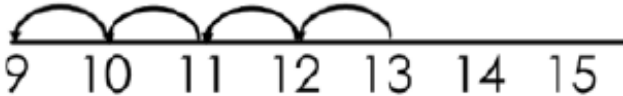
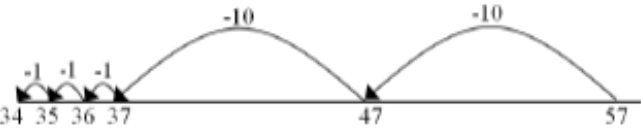
$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

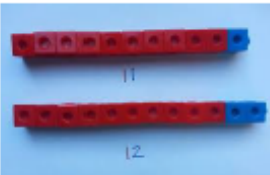
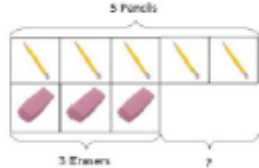
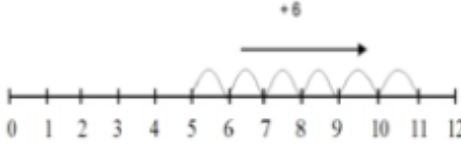
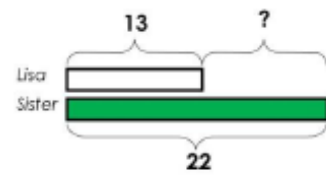
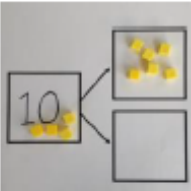
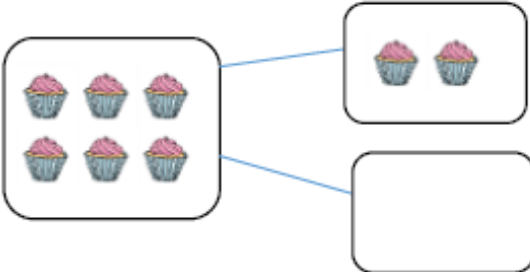


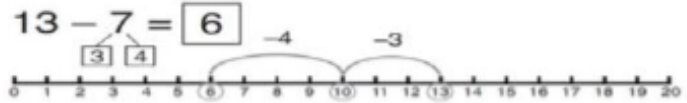
$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

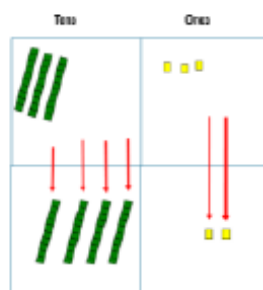
$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>6 - 2 = 4</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>15 - 3 = 12</p>	<p>18 - 3 = 15</p> <p>8 - 2 = 6</p>
<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>13 - 4</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

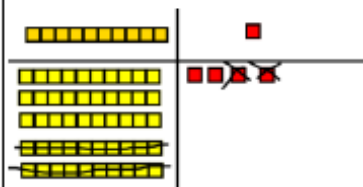
<h3>Find the difference</h3>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  <p>Draw bars to find the difference between 2 numbers.</p>	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<h3>Part Part Whole Model</h3>	 <p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<h3>Make 10</h3>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	<p>$13 - 7 =$ 6</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$\begin{array}{r} 47 - 24 = 23 \\ \begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array} \end{array}$$

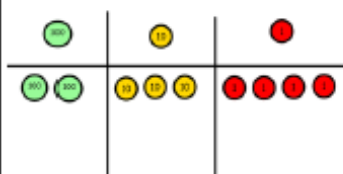
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



Calculations

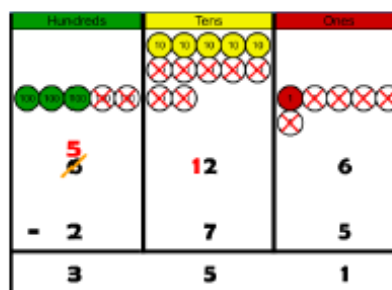
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

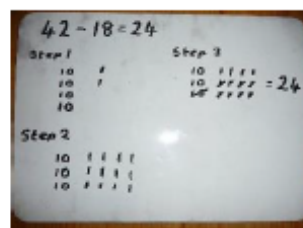


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method

and knows when to exchange/regroup.

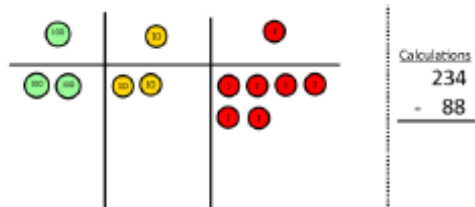
$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{r} 800 + 30 + 6 \\ - 200 + 50 + 4 \\ \hline 500 + 80 + 2 \end{array} \end{array}$$

Children can start their formal written method by partitioning the number into clear place value columns.

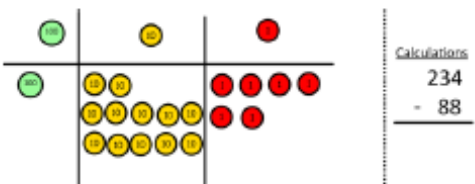
$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{r} 700 + 20 + 8 \\ - 500 + 80 + 2 \\ \hline 100 + 40 + 6 \end{array} \end{array}$$

Moving forward the children use a more compact method.

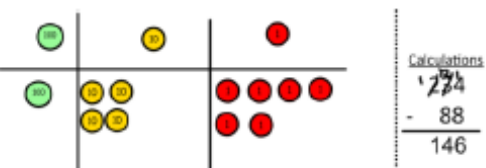
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Multiplication and Division

Vocabulary

Abstract, arrange, array, base, ten, calculation, combine, concrete, counters, double, equal, equal groups, exchange, groups of hundreds, lots of, multilink, multiply, numicon, number, facts, number line, number track, ones, partition, pictorial, place value, PPW – part-part-whole, product, record, repeated addition, tens, thousands

National Curriculum - Statutory Requirements

Year One – Pupils should be taught to:-

- * solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Year Two – Pupils should be taught to:-

- * recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- * calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs
- * show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- * solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year Three – Pupils should be taught to:-

- * recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- * write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- * solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year Four – Pupils should be taught to:-

- * recall multiplication and division facts for multiplication tables up to 12×12
- * use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- * recognise and use factor pairs and commutativity in mental calculations
- * multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- * solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Year Five – Pupils should be taught to:-

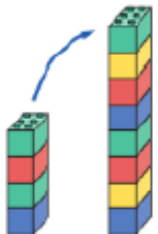

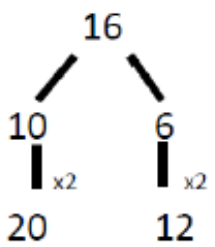




- * identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- * know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- * establish whether a number up to 100 is prime and recall prime numbers up to 19
- * multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- * multiply and divide numbers mentally drawing upon known facts

- * divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- * multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- * recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- * solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- * solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- * solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Year Six – Pupils should be taught to:-

- * multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- * divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- * divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- * perform mental calculations, including with mixed operations and large numbers
- * identify common factors, common multiples and prime numbers * use their knowledge of the order of operations to carry out calculations involving the four operations
- * solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- * solve problems involving addition, subtraction, multiplication and division
- * use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Counting in multiples	  <p>Count in multiples supported by concrete objects in equal groups.</p>	  <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Repeated addition

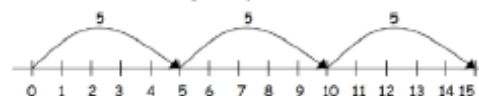


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$$5 + 5 + 5 = 15$$

Write addition sentences to describe objects and pictures.



$$2 + 2 + 2 + 2 + 2 = 10$$

Arrays- showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.



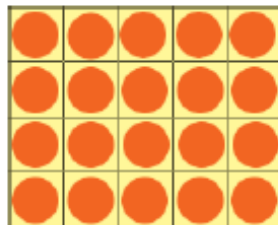
$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$



Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

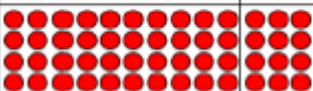

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

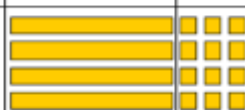

Grid Method

Show the link with arrays to first introduce the grid method.

x	10	3
4		

4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

x	T	U
4		













4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.













Calculations
4 x 126









Fill each row with 126.

Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.

Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

24	x	3	=	72
x	20	4		
3	00	0000		
	00	0000		
	00	0000		
	60	12		
		60		
		+ 12		
		72		

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$$210 + 35 = 245$$

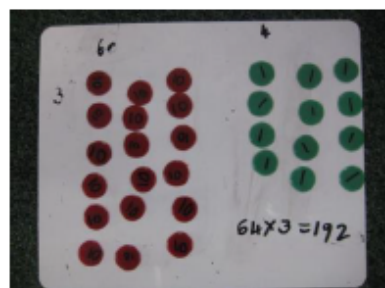
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

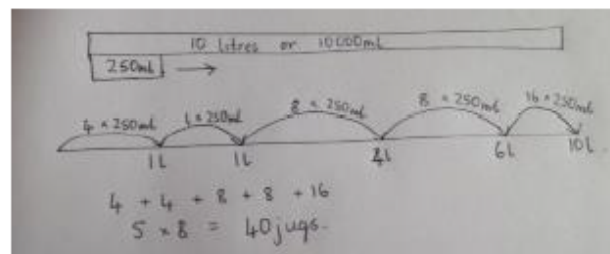
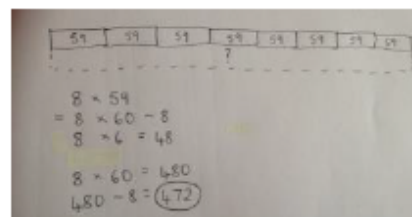
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



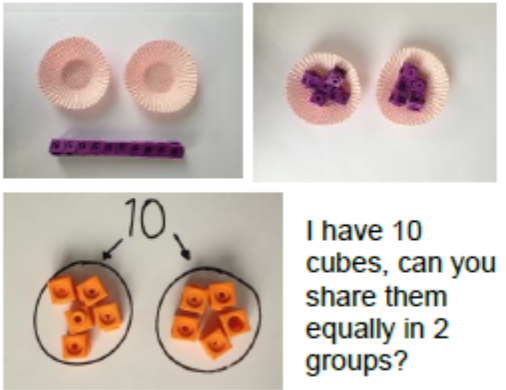
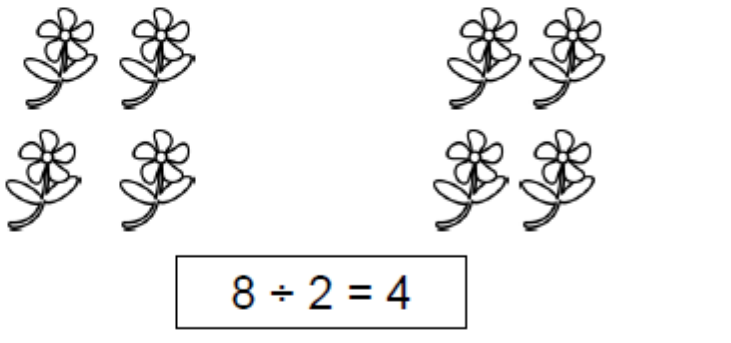
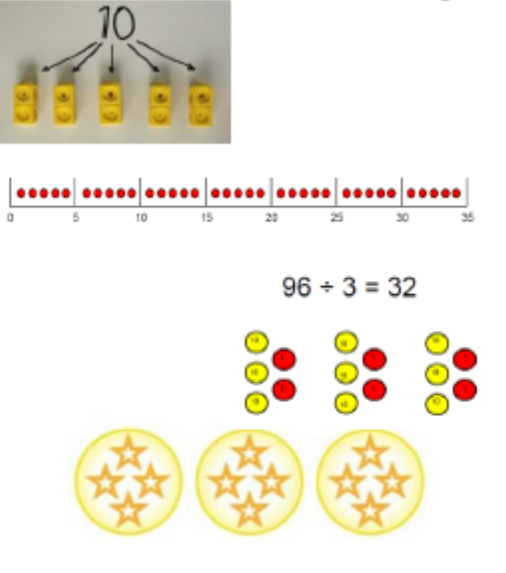
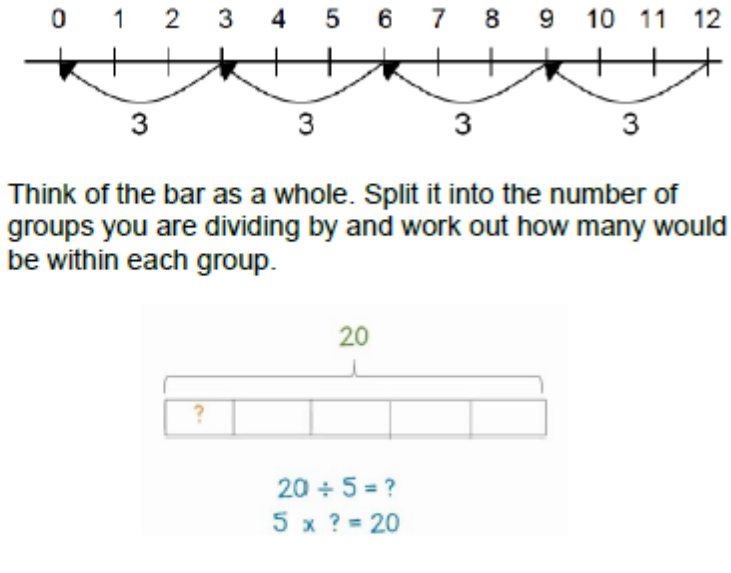
Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

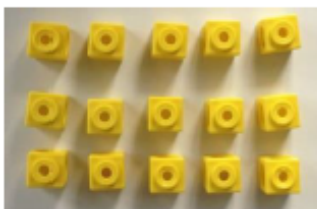
$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$

This moves to the more compact method.

$$\begin{array}{r}
 \begin{array}{ccc} 2 & 3 & 1 \end{array} \\
 1342 \\
 \times 18 \\
 \hline
 13420 \\
 10736 \\
 \hline
 24156
 \end{array}$$

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Sharing objects into groups</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  $96 \div 3 = 32$	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> $20 \div 5 = ?$ $5 \times ? = 20$	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

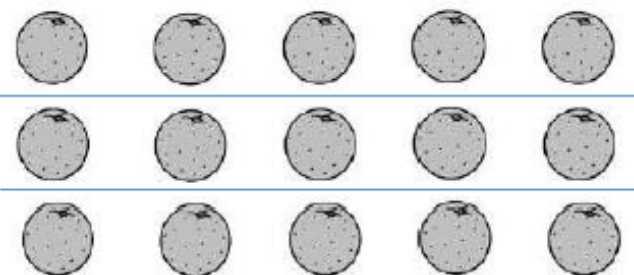
Division within arrays



Link division to multiplication by creating an array and thinking about the

number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



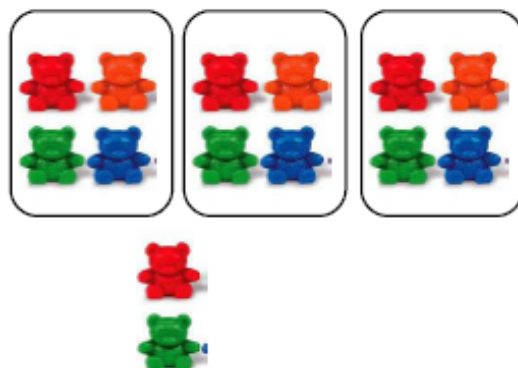
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



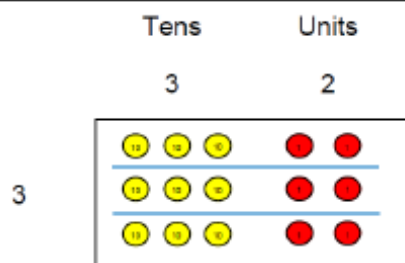
Draw dots and group them to divide an amount and clearly show a remainder.



Complete written divisions and show the remainder using r.

$29 \div 8 = 3 \text{ REMAINDER } 5$
 ↑ ↑ ↑ ↑
 dividend divisor quotient remainder

Short division



Use place value counters to divide using the bus stop method alongside



$42 \div 3 =$
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

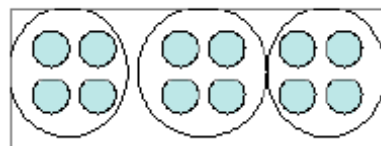


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$$

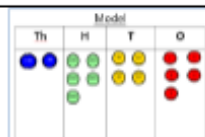
Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$

Long division



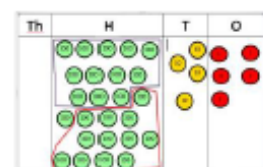
$2544 \div 12$
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds.



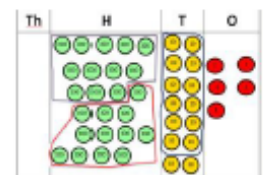
$$12 \overline{) 2544}$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



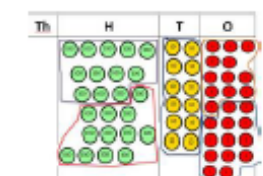
$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

$$\begin{array}{r} 0318r5 \\ 20 \overline{) 6365} \\ \underline{60} \\ 36 \\ \underline{20} \\ 165 \\ \underline{160} \\ 5 \end{array}$$